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WHAT WOULD YALE DO IF IT WERE TAXABLE?

The phenomenal success of Yale's endowment has been an inspiration to many investors. However, if Yale's endowment had to pay the same taxes as individual investors, its portfolio would be constructed very differently. This paper presents a simple model for incorporating tax considerations into a pre-tax asset allocation such as Yale's. With illustrative examples, we demonstrate the profound impact that taxes can have on optimal portfolio weights as well as the interplay between taxes and risk. Once taxes are included our model tends to lower allocations to tax-inefficient asset classes such as hedge funds and increase allocations to tax-efficient strategies. However, with optimal tax management, hedge fund allocation can still be preserved so long as their returns are uncorrelated with those of equity.

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David Swensen, the much-admired head of Yale's endowment, has done more than anyone to popularize the endowment model for institutional investors, with its focus on broad diversification into alternative asset classes. However, as a tax-exempt entity, Yale's endowment was designed without the need to consider taxes, and as a result is not directly applicable to taxable investors. In an ideal world David Swensen himself would instruct us on how to convert his endowment model to work as effectively in a taxable environment. While Swensen has not illustrated specifically how to do that, he has provided clues in his writing how the problem might be solved. Since the Yale endowment is so firmly grounded in modern portfolio theory, we can take advantage of one of its standard tools, but refitted to incorporate taxes.

In this paper, we describe how to derive implied pre-tax returns from an endowment like Yale's, how to convert those pre-tax returns to after-tax returns, and finally how to create a new asset allocation based on the reality faced by most wealthy investors – the fact that unlike Yale they have to pay taxes. This analysis will show how the Yale allocation before taxes looks nothing like its after-tax version. Investing in a taxable environment demands a completely new approach rather than relying on a direct transference of even a successful strategy like Yale's that was nonetheless designed for a tax-exempt setting. The result is an investment strategy that takes advantage of the Yale endowment model's diversification benefits but adapted to address the tax realities of the most affluent individual investors.

How the Endowment Model Became the Sexy Allocation for the Ultra-Affluent

Over the past twenty years, the wealth management industry that serves the most affluent individuals and families has adopted the institutional approach often defined as the endowment model. Wealth advisors for ultra-high net worth (UHNW) investors, those with tens of millions of dollars or more, have capitalized on potential advantages previously available only to large institutions. These include 1) the diversification benefits of higher allocations to alternative investments like hedge funds, private equity, natural resources and other illiquid assets, 2) sophisticated institutional-level research, models and concepts and 3) specialized consultants and databases to locate and measure more complex investment strategies. Advisors to the UHNW have further distinguished themselves from those serving the "mass affluent" as the proportion of wealth held by the UHNW has increased. This growing wealth concentration may be much discussed in the popular media, but not necessarily as an opportunity for a new breed of UHNW advisors.

Advisors have been drawn to Swensen's approach not only for diversification, but also for the extraordinary track record he has achieved at Yale. Widespread adoption of the endowment model for the UHNW reflects the desire to emulate Swensen's well-earned results. However, advisors to the UHNW face a critically important constraint in the form of taxes, an irrelevant consideration for exempt institutions such as Yale. Thus a simplistic implementation of the endowment model for taxable UHNW investors ignores

1) just how different an investment environment the taxable world can be and 2) how UHNW advisors will increasingly be forced by competitive pressure and higher tax rates to address the full tax impact.

To draw an analogy, picture the designers of an airport terminal facility, who for a terminal in Hawaii may include no external walls since the temperature tends to be mild year-round. However, engineering a terminal for New York City requires seriously different design concepts than would apply in Hawaii. If you were hiring an airport designer, you'd react with alarm if you were told that they were adapting their blueprint for a terminal in Hawaii to one in New York with just a few minor adjustments.

Most UHNW advisors do include some tax effects when analyzing portfolios, and they assure their clients that it's part of choosing the best strategies. However, the optimal approach requires incorporating tax effects from the very beginning, i.e. during the portfolio design stage. Most investors already understand, for example, simple tax concepts like municipal bonds offering a tax advantage over taxable bonds. However, far fewer pay any serious attention to the often significant differences between pre-tax and after-tax returns for equities, hedge funds, real assets and other asset classes, which requires measuring the tax impact, something few advisors actually do. Capitalizing on the diversification benefits from the endowment model requires bringing a Yale-like sophistication to after-tax analysis, modeling and measurement rather than just slapping the same asset allocation onto portfolios of taxable investors. Some recent trends in the investment world make this sophisticated approach a client-driven necessity: the recent jump in top-bracket tax rates, increased investor focus on taxes and a growing awareness that fiduciary duty includes assessing tax impact on a client's wealth.

Advisors to the UHNW face two challenges when implementing a more tax-efficient portfolio design. First, the complexity of tax law and its variation over time requires some assumptions to be made. However, the inclusion of broad tax effects across asset classes can be quite simple, e.g. for equities, index funds are going to be much more tax efficient than active management as a general rule. Thus it's important not to let the perfect be the enemy of the good when assessing the level of precision necessary in estimating tax impact. Furthermore, UHNW investors will increasingly expect rigorous tax analysis as part of what they pay their advisors for.

Second, the advisory world potentially faces a subtle conflict of interest in wanting to be perceived as adding value with investment products that may not look as good after-tax as they do pre-tax. Raising client awareness about the tax penalty can prove challenging due to not only the complexity of the subject but also the pall it can cast on an otherwise exciting story for some of the most glamorous strategies. This agency problem reflects the asymmetry of investors paying the cost of advisor-designed portfolios that don't fully incorporate taxes. As Dan diBartolomeo puts it,

> Although it is clear to everyone in the industry that after-tax returns are what economically matter to taxable investors, many firms continue to take the view that "we can ignore taxes entirely as long as our competitor managers do the same." In our view, this position of intentionally providing less than the best available services to separate account clients is unsustainable in the long run, and borders on a breach of fiduciary responsibility.[1]

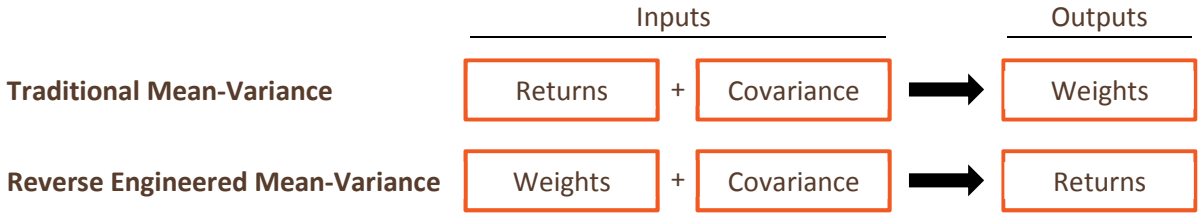
To make sure that the tax component gets incorporated into asset allocation from the design stage, UHNW advisors can use a simple mean-variance optimizer, which will need some minor adjusting for a taxable world. As UHNW investors become increasingly sophisticated about the impact of taxes (at much higher rates since 2013), their advisors face competitive pressure to incorporate some sort of measurement tool that can include the effect of taxes on asset allocation.

Ferreting Out Yale's Expected Returns

To answer the question of what Yale would do if it were taxable, we'll need to measure how Yale's tax-exempt asset allocation might change if its portfolio were subject to taxes. First we start with Yale's pre-tax expected returns, which we'll calculate[2] using a modified version of the traditional mean-variance optimizer. Traditionally, the most widely used application of the mean-variance optimizer by wealth managers has required forecasting inputs for all potential assets classes: expected return, risk and correlation. In fact, the last two can be derived from the covariance matrix, so it's really just two sets of inputs, return and covariance.

In the traditional mean-variance model, we need to know asset class expected returns in order to determine the optimal weights of the asset allocation. However, if we know the weights but not the expected returns, we can simply solve for a different output, as shown in Figure 1.

Figure 1: Two Applications of a Mean-Variance Optimizer



The reverse engineering approach was first proposed by William Sharpe in 1974,[3] although others have expanded on it since then.

Given that Yale makes public the asset allocation of its endowment, we can apply reverse engineering optimization to back into the endowment's expected returns. Both

inputs required can be accessed from publicly available information, the weights from the annual report of the Yale endowment[4] and the covariance calculated from benchmark indices in Morningstar Principia.[5] Whether or not these implied returns reflect Yale's or any other investor's true expectations, mathematically they are the only pre-tax returns that make the asset allocation weights fit with the presumed benchmarks and historical covariance. For advisors who follow the Yale allocation with interest, this calculation of the implied returns can provide fascinating insight into the interplay of expected risk and return assumptions used by David Swensen and his team, i.e. what expected returns are consistent with their asset allocation.

Table 1 shows the most recent Yale asset class weights, with foreign and U.S. equities collapsed into one asset class, along with the implied asset class returns from the reverse engineering.

Table 1: Asset Class Weights and Implied Returns for Yale Endowment

Asset Class	Benchmark	Yale Pre-Tax Weight	Implied Pre-Tax Return
Absolute Return	DJ Credit Suisse Hedge Fund	17.8%	2.2%
World Public Equity	MSCI All Country World	15.7%	9.7%
Bonds	Barclays Aggregate	4.9%	1.5%
Natural Resources	Goldman Sachs Natural Resources	7.9%	10.7%
Real Estate	DJ US Real Estate	20.2%	12.3%
Private Equity	Russell 2000	32.0%	11.7%
Cash	US Treasury Bills 0-3 Month	1.5%	1.5%

Sources: Weights from annual report of the Yale Endowment as of December 31, 2013; returns calculated by Aperio Group using reverse optimization model described herein. Past performance is not a guarantee of future returns. The performance above is hypothetical and does not represent actual performance of the Endowment or any portfolio. Please refer to important disclosures at the end of this report.

These implied returns may come as a shock, especially the low 2.2% return for absolute return (hedge funds). That implied return does not suggest that any consensus opinion of the expected return for hedge funds is only 2.2% or that Yale's investment experts expect that exact return. All it implies is that based on the historical covariance among those assets, a hedge fund return of 2.2% is the only one mathematically consistent with a reverse-engineered mean-variance analysis using the weights of the Yale endowment. The weights and covariance can produce no other answer. The only ways for the expected return for hedge funds to be higher would be if the Yale weight were higher or the historical covariance were different.

Investors may disagree with that expected return, but using a reverse-engineered model requires a consistency between the asset class weights and the expected returns, just as in the standard version expected returns and an historical covariance generate just one set of weights. A different set of benchmarks for those assets or a different time period for the calculation of the covariance could produce equally valid results. The

point here is not that these are the most accurate expected returns, but rather that another set of expected returns is possible only with different inputs. One motivation for relying on reverse-engineered implied returns in this paper is to avoid controversy around estimated expected returns, since most investors will have their own views.

Beyond the expected returns for hedge funds, investors may disagree on the choice of hedge fund index proxy since correlation with equity markets varies so widely across hedge fund benchmarks.[6] Table 1 reflects a hedge fund benchmark with a very low correlation with equity markets, which is one reason why the implied expected return is so low for the Absolute Return asset class. Later we'll look at the impact of choosing a different hedge fund benchmark.

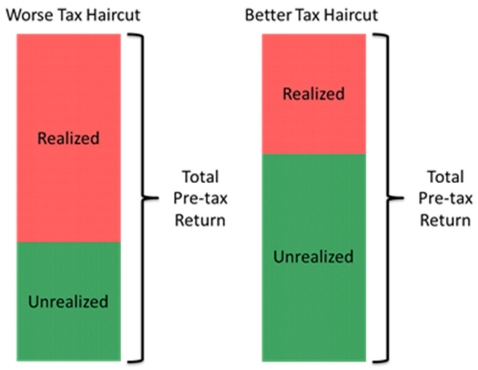
For investors who seek to modify specific asset class return forecasts to reflect their own expectations rather than relying on the implied returns from the Yale endowment, the well-known Black-Litterman model from 1992[7] expanded on Sharpe's original model. In Black-Litterman, investors can adjust expected returns up or down in order to incorporate their own views. Such tinkering with expectations can prove quite fruitful and informative in an applied portfolio analysis, but it's beyond the scope of this paper.

The Painful Yet Often-Ignored Tax Impact

Now that we have estimated the expected pre-tax returns implied by the Yale asset allocation, we'll need to adjust those returns to reflect the tax impact on various asset classes. That tax impact, which we'll call the "tax haircut," will convert pre-tax total return into after-tax total return. Pre-tax return can be divided into two main components, the unrealized portion and the realized portion, which can include ordinary income, dividends, or realized short- and long-term gains. Note that the risk data are based on pre-tax returns and have not been adjusted in order to reflect risk or correlation based on after-tax returns.[8]

For taxable investors, the higher the proportion of unrealized return, the more the investor can control the timing of the tax haircut, and in some situations avoid income tax entirely, e.g. with assets passed through an estate or donated to charities. With unrealized gains, an investor determines when, if ever, to pay income tax on pre-tax return, whereas with realized gains in a managed account, the investor's manager may determine the timing, quantity and rate (long vs. short) applicable to the tax liability. The fact that investors and not the managers pay the tax liability on realized returns may help explain why taxes are so often ignored as part of the asset allocation discussion beyond an allocation to municipal bonds. Figure 2 shows how for the same pre-tax return, an investment with a larger proportion of realized gain will mean higher taxes (and thus a lower after-tax return) versus a more tax-efficient investment with less of the pre-tax return subject to the tax penalty.

Figure 2: Breakdown of Realized and Unrealized Return



The above chart is shown for illustrative purposes only.

The magnitude of the tax haircut of course varies greatly across asset classes, but it also can vary a great deal within one specific asset class. Described below are the issues in estimating the tax haircut for major asset classes.

1. Public Equities

In the public equity space an investor can choose three different investment approaches: 1) traditional active management, 2) traditional indexing or 3) tax-advantaged indexing, also known as tax loss harvesting. For equity strategies, taxable income can come from either dividends or the realization of gains by an investor or the investor's manager. Active management on average tends to incur the worst tax haircut. As an historical example, we gathered data for all active U.S. equity funds from the Morningstar Principia database over the eighteen-year period ending June 30, 2013. We assumed that the dividend distribution would be the same for all three types of public equity strategies, i.e. all will be paying dividends comparable to the entire stock market, in this case as represented by the Russell 3000. If all three pay similar dividends, that leaves only the breakdown between realized and unrealized capital gain. Over that period, 76% of the total capital appreciation from actively managed funds was distributed as taxable income to investors. Of course that percentage will vary over time, but here we'll treat it as representative of the tax haircut of active management, rounding the realized portion down to 70% per year, and applying the same haircut to global equities.

Turning from active management to traditional indexing, while the dividends are presumed to remain the same, the realized gain component tends to be much smaller or zero. In our analysis it's presumed to be 0%, reflecting the fact that for the many efficiently managed broadly diversified large index funds, capital gains can be kept very low. Furthermore, many ETFs have been able to take advantage of the tax rules around redemptions of units to avoid capital gain distributions.

Finally, investors can choose a third version of equity strategies known as tax-advantaged indexing or tax loss harvesting, which seeks to track a benchmark pre-tax, albeit with higher tracking error than a pure index fund, while generating a tax benefit from realizing losses. Such a strategy can actually go further in improving the tax haircut than index funds, which cannot pass through losses by law. Tax-advantaged indexing can generate a positive impact on after-tax returns, assuming that a taxable investor has realized gains elsewhere that can be offset with losses. For a federal-only taxable investor in the top tax bracket, that benefit can translate into a gain as high as 1.92% per year for assets passed through an estate or to a charity. The benefit can still be as high as 1.03% for investors planning to liquidate at the end of a ten-year holding period.[9] Such gains from what is effectively tax deferral can be earned over longer periods of time, as long as twenty or thirty years, even when loss harvesting opportunities diminish in later years.[10] In states with high tax rates, the benefit is magnified.[11]

2. Bonds and Cash

   It is straightforward to estimate tax haircuts for bonds or cash, which will be assumed to have 100% of total return distributed as taxable ordinary income. No capital gains are assumed. When the Yale portfolio is analyzed as tax-exempt, the bond benchmark used as a proxy in the asset allocation is the taxable Barclays Aggregate index; when the Yale portfolio is treated as taxable, then the bond benchmark is the tax-exempt Barclays Municipal, with all return treated as exempt.

3. Absolute Return

   Estimating a tax haircut for strategies such as hedge funds presents a challenge in that public after-tax data are not readily available. Based on an informal survey of a select group of UHNW wealth managers, this analysis will assume the same haircut on gains for hedge funds as for actively managed equities.

4. Natural Resources

   Investments in natural resources also present the problem of very limited public data on the magnitude of the tax haircut. Furthermore, the method of capturing exposure to natural resources can significantly affect the tax haircut. Futures-based natural resource strategies may face significant tax penalties, while direct ownership can be highly tax-efficient due to preferential tax treatment of certain investments in timber or energy, for example.[12] For this analysis, it is assumed that 30% of gains for natural resources are realized, with no ordinary income assumed. In reality, investors face a very wide range of tax treatments for natural resource investments, but it will be defined in this analysis as relatively tax-efficient, certainly compared to active equity or hedge funds.

5. Real Estate

   Similar to natural resources, real estate investing offers structures with a wide range of tax treatment. Direct investment or pass-through pooled vehicles such as partnerships can offer highly advantageous tax treatment, again taking advantage of a number of tax benefits for real estate investing. However, ownership through real estate investment trusts, or REITs, do not offer the same tax benefits. In fact they can be tax inefficient since REIT dividends do not qualify for the beneficial dividend tax rate, instead being subject to the highest rate for ordinary income. For this analysis the assumption will be 30% of total return as ordinary income with no realized gains.

6. Private Equity

   While private equity can offer tax deferral at capital gains rates, many investors capture this asset class through funds, which do distribute gains and other income to investors. Again the same relatively tax-efficient assumptions have been applied, with 30% of gain realized each year, all of it long-term. Dividend assumptions reflect the same as for public equities.

All of the assumptions about taxation by asset class can be seen in Table 2. Long-term capital gains are assumed to comprise two-thirds of total realized capital gains and short-term the remaining one third, except for gains for private equity, assumed to be all long-term.[13]

Table 2: Estimation of Tax Penalty Across Asset Classes

	% of Total Return from					
Asset Class	**Ord. Income**	**Realized Short Gains**	**Realized Long Gains**	**Unrealized**	**Dividend Return**	**Tax Benefit**
Absolute Return	0%	23%	47%	30%	0.0%	0.0%
Equity, Active*	0%	23%	47%	30%	2.0%	0.0%
Equity, Indexed*	0%	0%	0%	100%	2.0%	0.0%
Equity, Tax-Adv.*	0%	0%	0%	100%	2.0%	1.9%
Taxable Bonds	100%	0%	0%	0%	0.0%	0.0%
Municipal Bonds	0%	0%	0%	0%	0.0%	0.0%
Nat. Resources	0%	10%	20%	70%	0.0%	0.0%
Real Estate	30%	0%	0%	70%	0.0%	0.0%
Private Equity*	0%	0%	30%	70%	2.0%	0.0%
Cash	100%	0%	0%	0%	0.0%	0.0%

*For equity strategies, the % of Total Return applies to only the non-dividend portion of total return, i.e. gains. For all public and private equity strategies, dividends are assumed to be taxed at qualified dividend rates applied to a constant 2.0% dividend yield.

The above estimates are hypothetical and are not based on specific individual investments. Please refer to important disclosures at the end of this report.

The tax haircut calculations shown in Table 2 reflect the assumptions for all asset classes based only on the ongoing tax effect over time.[14] Not included in the assumptions are the effects of tax impact upon liquidation, which could lower the distinction among asset classes. The assumptions shown in Table 2 reflect disposition through charity or estate, where a basis step-up would eliminate the effect of taxation on liquidation. Estate taxes could further erode after-tax wealth, but this paper will focus exclusively on the effects of income taxes.

These tax haircut estimates represent just one of many valid sets of assumptions for a top-bracket taxpayer. Investors may face different tax outcomes due to a particular strategy or tax situation. The goal here is not to derive the unattainable one single "true" set of tax assumptions, but rather to focus on the transparency of measuring and estimating actual tax impact rather than assuming it away as too complicated or irrelevant. Investors and their advisors appropriately focus on the fees and transaction costs that can diminish returns. However, at the same time they can inappropriately shrug off a tax impact that at rates as high at 52%[15] can potentially wipe out over half of pre-tax return, a reduction of wealth that can dwarf other costs.

How a Great Pre-Tax Asset Allocation Can Look So Much Worse After-Tax

Now we have both 1) a model to calculate expected pre-tax returns based on existing weights like those of the Yale endowment and 2) estimates of tax haircuts across asset classes. These tools will make it easier to measure and analyze the subtle interplay of the tax penalty and correlation among asset classes. The analysis of what Yale would do if it were taxable consists of the following three steps using the tools described above:

1. Based on the reverse-engineered version of the mean-variance optimizer, calculate the implied pre-tax returns for a portfolio given the existing asset allocation.
    - Inputs: pre-tax asset allocation weights and pre-tax covariance.
    - Output: expected pre-tax returns.

2. Apply the tax haircuts to adjust the pre-tax returns to reflect the actual after-tax experience of a taxable top-bracket investor instead of a tax-exempt endowment.
    - Input: expected pre-tax returns.
    - Output: expected after-tax returns.

3. Use the traditional mean-variance optimizer to generate an ideal set of weights for the same asset classes but for a taxable investor rather than a tax-exempt endowment.
    - Inputs: expected after-tax returns and pre-tax covariance.
    - Output: after-tax asset allocation weights.

Table 3 shows on the left the implied returns, both pre- and after-tax, and on the right the adjusted weights once the Yale portfolio becomes subject to taxes. The variations reflect three different assumptions about how the portfolio includes exposure to public equities. As can be seen in the table, it turns out that the choice of strategy for public equities can lead to very different overall asset allocations beyond just that one asset class.

Table 3: After-Tax Returns and Weights for Yale (low correlation HF index)

	Returns			Weights			
Asset Class	**Yale P/T Implied Return**	**Yale A/T Implied Return**	**Yale Tax Haircut**	**Yale P/T Weight**	**Yale A/T: Active Equity**	**Yale A/T: Indexed Equity**	**Yale A/T: Tax-Adv. Equity**
Absolute Return	2.2%	1.7%	-0.5%	17.8%	0.0%	0.0%	11.9%
Equity, Active	9.7%	7.5%	-2.2%	15.7%	0.0%	0.0%	0.0%
Equity, Indexed	9.7%	9.2%	-0.5%	0.0%	0.0%	45.6%	0.0%
Equity, Tax-Adv.	9.7%	11.1%	+1.4%	0.0%	0.0%	0.0%	26.4%
Bonds*	1.5%	0.9%	-0.7%	4.9%	35.0%	25.8%	14.5%
Nat. Resources	10.7%	9.7%	-1.0%	7.9%	12.2%	0.0%	5.2%
Real Estate	12.3%	10.7%	-1.6%	20.2%	13.9%	9.0%	2.8%
Private Equity	11.7%	10.4%	-1.3%	32.0%	38.9%	19.6%	39.1%
Cash	1.5%	0.8%	-0.7%	1.5%	0.0%	0.0%	0.0%

*Bonds are assumed to be taxable for the pre-tax allocation and tax-exempt municipals for the after-tax portfolios. All distributions from tax-exempt bonds are assumed to be tax-exempt, i.e. possible capital gains are ignored.

Note: P/T = Pre-Tax, A/T = After-Tax. The above weights and returns are hypothetical and are not based on actual investments. Cash has been constrained in optimization to show a weight of zero. Please refer to important disclosures at the end of this report.

How would the asset allocation of Yale's endowment need to change if it were taxable? The answer depends greatly on how public equity as an asset class is implemented in the portfolio. Each version of implementing an after-tax portfolio is explained in more detail below.

1. The Yale After-Tax with Active Equity (yellow column) shows the weights of the Yale portfolio with the equity allocation available only through active strategies, thus subject to the tax haircut of 70% of total capital gain being realized each year. This version presupposes an investor who uses only active management for public equities. Given that an investor now has to pay taxes on what had been a tax-exempt portfolio, suddenly active equity and absolute return (hedge fund) allocations become far less attractive due to the significant tax haircut crimping what had been

more attractive pre-tax returns. Municipal bonds earn a high allocation due to lower risk, while the high weight in private equity reflects its relative tax efficiency. Private equity, reflecting Yale's choice of the Russell 2000 as the appropriate benchmark, provides the least painful method of achieving equity exposure once the tax drag is applied.

2. The Yale After-Tax with Indexed Equity (blue column) shows the portfolio under the same assumptions used in scenario 1, but with indexed equities now available as well. Given that the tax haircut on capital gains does not apply in this equity strategy, the portfolio composition changes significantly to weight equities and bonds heavily, with a smaller portion in private equity.

3. The Yale After-Tax with Tax-Advantaged Equity (green column) shows the portfolio with access to the same indexed strategy but adds tax-advantaged equities as well. The allocation to the tax-advantaged equity strategy is limited by the amount of capital gain generated by other asset classes, allowing only enough to match what can be offset directly within the portfolio each year, based on the tax haircut assumptions defined earlier. In this scenario we see that the hedge funds, which had not been attractive when only active or standard indexing was available, now become attractive due to the capacity of the tax-advantaged strategy to offset the presumed tax inefficiency of hedge funds.

What if we look beyond the Yale endowment to other portfolios appropriate for different investor situations? Appendix I shows results for the average of a broad range of university endowments, based on data from the National Association of College and University Business Officers (NACUBO). The NACUBO data show a lower allocation to more illiquid asset classes like private equity, not surprising given the Yale endowment's size and emphasis on capturing a perceived illiquidity premium. Other than that difference, though, the effect of adjusting for taxes for NACUBO shows similar and intuitive results: the more tax-inefficient asset classes no longer provide the same value once an investor has to pay taxes.

Turning now from tax-exempt sample portfolios to those of taxable UHNW investors, we'll look at a representative example of what an advisor to the UHNW might allocate before and after taxes. The sample portfolio analyzed in Table 4 represents just one plausible allocation among many for a portfolio for the UHNW and does not reflect any empirical data.

Table 4: After-Tax Returns and Weights for Sample UHNW Advisor (low correlation HF index)

	Returns			Weights			
Asset Class	**Sample Advisor P/T Implied Return**	**Sample Advisor A/T Implied Return**	**Sample Advisor Tax Haircut**	**Sample Advisor P/T Weight**	**Sample Advisor A/T: Active Equity**	**Sample Advisor A/T: Indexed Equity**	**Sample Advisor A/T: Tax-Adv. Equity**
Absolute Return	2.5%	1.9%	-0.5%	25.0%	13.8%	11.3%	24.5%
Equity, Active	10.9%	8.4%	-2.5%	30.0%	0.0%	0.0%	4.4%
Equity, Indexed	10.9%	10.4%	-0.5%	0.0%	0.0%	51.0%	0.0%
Equity, Tax-Adv.	10.9%	12.3%	+1.4%	0.0%	0.0%	0.0%	24.6%
Bonds*	1.7%	0.9%	-0.7%	20.0%	42.3%	34.1%	23.2%
Nat. Resources	12.0%	10.8%	-1.1%	5.0%	13.1%	0.0%	5.7%
Real Estate	12.9%	11.2%	-1.7%	10.0%	8.0%	2.5%	0.0%
Private Equity	12.1%	10.8%	-1.3%	10.0%	22.9%	1.1%	17.5%
Cash	1.5%	0.8%	-0.7%	0.0%	0.0%	0.0%	0.0%

*Bonds are assumed to be taxable for the pre-tax allocation and tax-exempt municipals for the after-tax portfolios. All distributions from tax-exempt bonds are assumed to be tax-exempt, i.e. possible capital gains are ignored.

Note: P/T = Pre-Tax, A/T = After-Tax. The above weights and returns are hypothetical and are not based on actual investments. Cash has been constrained in optimization to show a weight of zero. Please refer to important disclosures at the end of this report.

As with the Yale portfolio, active equity brings such a high tax penalty that the optimal portfolio avoids it entirely, reallocating to hedge funds and other alternatives. Hedge funds still receive some allocation, but much less than in the pre-tax version, except in the case where the tax-advantaged equity can provide some offset to the tax penalty. The value of hedge funds, however, still depends very much upon their correlation to equity markets, as we'll see in the next section.

How Taxes Amplify the Impact of Correlation

The results seen above show the changes to asset class weights when a tax-exempt portfolio becomes subject to the ravaging effects of taxation. The less tax-efficient the asset class, the lower the likelihood it will maintain as high an allocation in the taxable portfolio as it had in the tax-exempt. Beyond just the tax haircut, though, the correlation among asset classes affects the outcome significantly. For example, there exist substantial differences for hedge fund (absolute return) benchmarks depending on the chosen index. In the previous section, the hedge fund asset class reflects the performance history of the Dow Jones Credit Suisse Hedge Fund index, which had relatively low correlation with equities over the time period studied, monthly data from

December 31, 1998 through June 30, 2013. Other hedge fund indices have shown much higher correlation with equity returns over the same time period, as shown in Table 5.

Table 5: Correlation for Hedge Fund Indices

Index Benchmark for Absolute Return (hedge funds)	Correlation with Equities*
DJ Credit Suisse Hedge Fund (low correlation)	0.14
HFRI Fund Weighted Composite (high correlation)	0.88

*As measured by the MSCI ACWI

The hedge fund correlation with equities materially changes the outcome. We saw in Table 3 how hedge funds disappear from the allocation once taxes are introduced, reappearing only when the constrained tax-advantaged strategy allows the portfolio to benefit from the potential risk benefits of hedge funds enough to offset the tax haircut. Thus when equity strategies are active or traditionally indexed in the model, the tax penalty on hedge funds outweighs any correlation benefit unless the tax effect can be mitigated by combining it with the benefits of tax-advantaged equity. However, if a hedge fund strategy reflects the risk patterns of the HFRI index, with its higher correlation to equities, then the model never allocates anything to hedge funds no matter how public equities are implemented, as seen in Table 6.

Table 6: After-Tax Returns and Weights for Yale (high correlation HF index)

	Returns			Weights			
Asset Class	**Yale P/T Implied Return**	**Yale A/T Implied Return**	**Yale Tax Haircut**	**Yale P/T Weight**	**Yale A/T: Active Equity**	**Yale A/T: Indexed Equity**	**Yale A/T: Tax-Adv. Equity**
Absolute Return	4.2%	3.3%	-0.9%	17.8%	0.0%	0.0%	0.0%
Equity, Active	9.2%	7.1%	-2.1%	15.7%	0.0%	0.0%	0.0%
Equity, Indexed	9.2%	8.7%	-0.5%	0.0%	0.0%	45.6%	0.0%
Equity, Tax-Adv.	9.2%	10.6%	+1.4%	0.0%	0.0%	0.0%	25.0%
Bonds*	1.5%	0.9%	-0.7%	4.9%	35.0%	25.8%	25.6%
Nat. Resources	10.2%	9.2%	-1.0%	7.9%	12.2%	0.0%	5.8%
Real Estate	11.4%	9.9%	-1.5%	20.2%	13.9%	9.0%	2.3%
Private Equity	11.0%	9.8%	-1.2%	32.0%	38.9%	19.6%	41.3%
Cash	1.5%	0.8%	-0.7%	1.5%	0.0%	0.0%	0.0%

*Bonds are assumed to be taxable for the pre-tax allocation and tax-exempt municipals for the after-tax portfolios. All distributions from tax-exempt bonds are assumed to be tax-exempt, i.e. possible capital gains are ignored.

Note: P/T = Pre-Tax, A/T = After-Tax. The above weights and returns are hypothetical and are not based on actual investments. Cash has been constrained in optimization to show a weight of zero. Please refer to important disclosures at the end of this report.

Note that the expected pre-tax returns for absolute return strategies (hedge funds) are higher than they were assuming the low correlation benchmark previously shown in Table 3. However, under these conditions highly correlated hedge funds still cannot be justified in the presence of taxes because they offer so little diversification benefit to offset the tax haircut. When presented with this observation, one wealth manager commented, "Well of course I wouldn't pay the tax penalty to be in a tax-inefficient hedge fund strategy unless I were gaining the risk advantages of low correlation."

In terms of how the different asset classes are affected, we've already seen how the tax drag for hedge funds makes the model include them only when the correlation with equity is low and when the tax hit can be offset. Private equity often gets a boost when taxes are introduced, due to the assumptions about relative tax efficiency. However, if index equities are available, then the model optimizes, as seen for example in Table 6, to lower the exposure to private equity, which then rises again in the presence of tax-advantaged equity. That pattern reflects the high correlation between private and public equity, but tempered by the tax haircut.

Swensen's Genius Misapplied

When Swensen published his first book in 2000, *Pioneering Portfolio Management*, many UHNW advisors saw the benefits of capitalizing on the best of the endowment model: superior risk diversification, the illiquidity premium, and access to more rarified strategies, among other benefits. The UHNW investment world eagerly embraced the new approach as more sophisticated while sometimes glossing over the fact that it had been designed for a tax-exempt world. However, in his second book, *Unconventional Success*, Swensen explicitly recognizes the harm of ignoring taxes once you change environments:

> The management of taxable…assets without considering the consequences of trading activity represents a…little considered scandal. A serious fiduciary with responsibility for taxable assets recognizes that only extraordinary circumstances justify deviation from a simple strategy of selling losers and holding winners.[16]

For the taxable world, Swensen recommends a radically different approach from what he does in the tax-exempt world because he understands so well how harmful the tax haircut can be to after-tax wealth. In solving the tax challenge, he doesn't propose just a minor adjustment to asset allocation. Instead he focuses on considering the impact of all trading on a portfolio, which requires tools like those described in this study.[17] He also asserts that fiduciary responsibility compels a manager of taxable assets to seek a solution of selling losers and holding winners. For public equities, that "simple strategy" is effectively the tax-advantaged strategy described herein.

What are the lessons learned from evaluating the Yale endowment with these tools? First, the changes in asset allocation can be so dramatic that it's incumbent on the savvy investor to combine risk and taxes in the design stage of a portfolio. The tax effect cannot be layered in after the fact as some minor tweak, such as seeking slightly more tax efficient hedge funds versus tax-inefficient ones. That remains an admirable goal, but a truly optimal after-tax portfolio requires starting from scratch when assessing how risk and tax effects interact.

Second, in the after-tax version of the Yale endowment with only actively managed equity strategies, the optimizer includes no equity exposure at all. That might sound heretical to many managers, but the math says that if you're facing a tax haircut like the average for active equity funds, then it's not worth owning any equities at all if you believe in Yale's asset allocation. However, once you allow indexing, with its significantly reduced trading, then suddenly a good part of the portfolio looks like a simple 1960s-style 60%/40% stock and bond pension fund. Adding a tax-efficient equity strategy goes one step further by allowing the portfolio to hold more tax-inefficient assets like hedge funds and take advantage of their diversification benefits while offsetting some of the tax drag.

Third, correlations matter even more in the presence of taxes. In our analysis, hedge funds with high correlation to equity markets never get included in the Yale portfolio once taxes are introduced. This is an indication that there's just no place for highly inefficient tax strategies unless there's a big benefit from low correlation. Alternative strategies that are highly correlated with equities and also generate a lot of gains from trading can't survive in a portfolio designed to maximize after-tax returns.

The new framework and accompanying metrics can help investors and advisors pursue many of the benefits of the endowment model but now in a way that takes into account the harsh after-tax reality of the world of UHNW investors. To the question "What would Yale do?" the answer is to bring the analytic sophistication of a world-class investment shop like Yale's to taxable UHNW investors rather than trying to superimpose an approach designed for only the tax-exempt world. After all, truly adaptive geniuses are smart enough to look at a new environment from a completely different perspective and thus make decisions superior to merely applying the techniques that might have made them successful elsewhere.

Endnotes

[1] Dan diBartolomeo, "Northfield News," September 2010, p. 1, http://www.northinfo.com/Documents/390.pdf.

[2] Throughout this paper, we may use the term "calculate" to describe the math behind the generation of estimates of expected return or covariance based on historical data. In statistical parlance, these calculations produce only estimates, which should not be confused with precise and known values. When dealing in general with financial statistics or optimizers, a fair dose of humility helps offset our natural human hubris and tendency to think we know more than we actually do. While the analysis described in this paper can provide valuable insight, it does not tell us precisely what expected returns will be, but rather which estimated expected returns make the model work mathematically.

[3] William F. Sharpe, "Imputing Expected Security Returns from Portfolio Composition," *Journal of Financial and Quantitative Analysis*, Volume 9, Issue 03, June 1974, pp. 463-472.

[4] Yale University, 2013 annual report of the Yale Endowment, http://investments.yale.edu/images/documents/Yale_Endowment_13.pdf.

[5] Morningstar Principia, data as of June 30, 2013.

[6] For the correlation coefficient estimates across all benchmarks, see Appendix II.

[7] Fischer Black and Robert Litterman, "Global Portfolio Optimization," *Financial Analysts Journal*, Vol. 48, No. 5 (Sep.-Oct., 1992), pp. 28-43.

[8] It could be argued that for a taxable investor after-tax covariance is the most accurate measure, but for simplicity this paper makes no adjustment to pre-tax risk values. Average historical tax haircut data can be hard enough to estimate; monthly after-tax return streams for alternative asset classes are even more elusive.

[9] For detail on value added from a tax-advantaged strategy, see Patrick Geddes, "The Pursuit of After-Tax Returns, Indexed Exchange-Traded Funds vs. Indexed Separately Managed Accounts," *Investments & Wealth Monitor*, September/October 2011, pp. 38-43, with a copy available at http://www.aperiogroup.com/system/files/documents/iwm11sepoct_pursuitaftertaxreturns.pdf. The potential benefit from a tax-advantaged strategy reflects the highest applicable federal income tax rates as of December 31, 2013. For more details on the Monte Carlo simulations used to estimate the benefit of a tax-advantaged equity strategy, see Disclosure section.

[10] The harvesting opportunities do tend to diminish fairly significantly after ten years, but the time value of deferral can continue to add value even after much of the harvesting has already been realized. See the Disclosure section for more detail.

[11] Taxable investors in California, for example, can benefit as much as 2.40% per annum for portfolios passed through an estate and 1.20% for those liquidated at the end of a ten-year period. California marginal income tax rate is assumed to be 13.3% and fully deductible against federal income. See Disclosure section for additional details.

[12] Investors can also achieve commodity exposure through natural resource stocks, which can be managed with a high degree of tax efficiency.

[13] The assumption on breakdown between long and short gains was kept consistent across all asset classes except for private equity. For empirical results based on older data, see John Bogle, "The Economics of the Mutual Fund Industry: For Fund Investors…For Fund Managers," speech to Albright College, October 5, 1999, p. 5,

http://johncbogle.com/speeches/JCB_albright_10-99.pdf. Bogle's estimate from that time showed 65% long-term and 35% short-term.

[14] All tax adjustments are based on the highest tax bracket rates for U.S. federal income tax only. Those rates reflect tax law changes effective January 1, 2013, which include a top short-term gain rate of 39.6%, a long-term gain rate of 20.0%, an additional 3.8% tax on Net Investment Income (the Medicare tax) and an effective additional 1.2% rate increase due to the phase-out of deductions for top-bracket taxpayers (the Pease phase-out). All of these rates combine to 44.6% for short gains and ordinary income and 25.0% for long gains and dividend income. All dividend income has been assumed to come from qualified dividends. State taxes would make the tax haircut even more expensive, but the outcomes remain similar to what was calculated for the impact of only federal income tax.

[15] For the highest bracket in California, the marginal tax rate includes the 44.6% federal rate (as calculated in the previous endnote) plus the top California rate of 13.3%. The California liability is assumed to be deductible for federal tax purposes (conventional 39.6% plus 3.8% tax on Net Investment Income, though the latter is subject to some limitations on deductibility of state taxes). The California rate of 13.3% gross thus reduces to a net impact of (1 - .396 - .038)*.133 = 7.5%, which means a total liability of 52.1% when added to the federal rate of 44.6%.

[16] David F. Swensen, *Unconventional Success*, New York, Free Press, 2005, p. 217.

[17] Mean-variance optimizers can show how risk, return and taxes all combine together within a portfolio. Measuring how taxes change the asset allocation provides valuable insight into just how expensive the tax penalty can be for investments that trigger significant tax liability. However, investors should avoid viewing output from a mean variance optimizer as a definitive best solution. Investment decisions require judgment of many factors around return, risk and taxes, and the output from an optimizer should not be viewed as "the right answer." Furthermore, the assumptions that went into the particular analyses described in this paper may not match exactly a specific investor situation. The taxonomy of categorizing different investment strategies into asset classes helps streamline decision-making, but it also can lead to a certain amount of oversimplification. For example, the myriad of hedge funds means that a particular representative index may not serve as a satisfactory proxy for every hedge fund available to the UHNW investor. Similarly, while hedge funds are often perceived as relatively inefficient in terms of tax management, some funds may be quite efficient, while others do distribute significant taxable income, frequently at expensive tax rates applied to short-term gains. Nonetheless, neither the complexity nor lack of precise data should be construed as excuses to avoid serious analysis of how returns, risk and taxes combine within a portfolio.

Appendix I: Analysis of NACUBO Portfolio

Table A1: After-Tax Returns and Weights for NACUBO (low correlation HF index)

	Returns			Weights			
Asset Class	**NACUBO P/T Implied Return**	**NACUBO A/T Implied Return**	**NACUBO Tax Haircut**	**NACUBO P/T Weight**	**NACUBO A/T: Active Equity**	**NACUBO A/T: Indexed Equity**	**NACUBO A/T: Tax-Adv. Equity**
Absolute Return	2.1%	1.6%	-0.5%	12.0%	0.0%	0.0%	11.0%
Equity, Active	9.3%	7.1%	-2.1%	41.0%	0.0%	0.0%	9.7%
Equity, Indexed	9.3%	8.8%	-0.5%	0.0%	0.0%	65.7%	0.0%
Equity, Tax-Adv.	9.3%	10.7%	+1.4%	0.0%	0.0%	0.0%	26.7%
Bonds*	1.6%	0.9%	-0.7%	15.0%	44.0%	30.9%	21.9%
Nat. Resources	10.5%	9.5%	-1.0%	10.0%	20.5%	2.0%	11.8%
Real Estate	10.5%	9.1%	-1.4%	10.0%	8.1%	1.0%	0.0%
Private Equity	10.2%	9.1%	-1.1%	10.0%	27.5%	0.4%	19.0%
Cash	1.5%	0.8%	-0.7%	2.0%	0.0%	0.0%	0.0%

*Bonds are assumed to be taxable for the pre-tax allocation and tax-exempt municipals for the after-tax portfolios. All distributions from tax-exempt bonds are assumed to be tax-exempt, i.e. possible capital gains are ignored.

Note: P/T = Pre-Tax, A/T = After-Tax. The above weights and returns are hypothetical and are not based on actual investments. Cash has been constrained in optimization to show a weight of zero. Please refer to important disclosures at the end of this report. Alternatives are assumed to be weighted equally among natural resources, real estate and private equity, as NACUBO does not provide that breakdown. Source: Pre-tax weights from http://www.nacubo.org/; returns and after-tax weights calculated by Aperio Group.

The outcome for average endowments from NACUBO (National Association of College and University Business Offices) ends up with after-tax weights fairly similar to those for the Yale endowment, as shown in Table A1. The high tax penalty for active management prevents the NACUBO model from holding any equity at all when only active equity is available, as private equity provides a lot of the same exposure, due to the assumption that it has much lower turnover. The blue column shows how with equity provided through traditional indexing, the outcome looks very similar to the old pension standby of 60% stocks and 40% bonds, with a light sprinkling of alternatives. The green column shows constrained tax-efficient equity as an available strategy, which allows for much more alternatives due to the presence of tax-advantaged equity. In addition, it reflects the odd outcome of actively-managed equity selected over traditional indexing. That counterintuitive result reflects the fact that the estimated tax benefit from tax-advantaged equity happens to be larger than the penalty for active management, meaning that a taxable investor would be better off holding more active equity because it allows more tax-advantaged equity.

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Appendix II: Correlation Coefficients

List of Asset Classes

Proxy #	Asset Class	Index Benchmark
1	Low Corr. Absolute Return	DJ Credit Suisse Hedge Fund USD
2	High Corr. Absolute Return	HFRI Fund Weighted Composite Index
3	World Equity - Active	MSCI ACWI NR USD
4	World Equity - Tax Neutral	MSCI ACWI NR USD
5	World Equity – Tax-Advantaged	MSCI ACWI NR USD
6	Fixed Income	Barclays Aggregate Bond TR
7	Municipal Bonds	Barclays Municipal
8	Natural Resources	Goldman Sachs Natural Resources
9	Private Equity	Russell 2000 TR
10	Real Estate	DJ US Real Estate TR USD
11	Cash	ML US Treasury Bills 0-3 Mon TR USD

Estimates of Correlation Coefficients for Data from December 31, 1998 through June 30, 2013

	1	2	3	4	5	6	7	8	9	10	11
1	1.00	0.27	0.14	0.14	0.14	-0.15	-0.01	0.10	0.14	0.14	-0.08
2	0.27	1.00	0.88	0.88	0.88	-0.02	0.10	0.80	0.80	0.56	-0.32
3	0.14	0.88	1.00	1.00	1.00	0.02	0.07	0.80	0.87	0.72	-0.21
4	0.14	0.88	1.00	1.00	1.00	0.02	0.07	0.80	0.87	0.72	-0.21
5	0.14	0.88	1.00	1.00	1.00	0.02	0.07	0.80	0.87	0.72	-0.21
6	-0.15	-0.02	0.02	0.02	0.02	1.00	0.63	-0.04	-0.10	0.17	0.13
7	-0.01	0.10	0.07	0.07	0.07	0.63	1.00	0.00	0.00	0.19	-0.02
8	0.10	0.80	0.80	0.80	0.80	-0.04	0.00	1.00	0.69	0.50	-0.25
9	0.14	0.80	0.87	0.87	0.87	-0.10	0.00	0.69	1.00	0.76	-0.14
10	0.14	0.56	0.72	0.72	0.72	0.17	0.19	0.50	0.76	1.00	-0.05
11	-0.08	-0.32	-0.21	-0.21	-0.21	0.13	-0.02	-0.25	-0.14	-0.05	1.00

Source: Return data from Morningstar Principia for all indices except for proxy #2, from HFRI. For more information on HFRI indices, see https://www.hedgefundresearch.com/mon_register/index.php?fuse=login_bd&1397845036.
Correlations were calculated from monthly pre-tax total return data.

Disclosure

The performance reflected in the tables and charts in this report are hypothetical, shown for illustrative purposes only, and not based on actual investments. Furthermore, they do not reflect the deduction of any management fees or transaction costs, which would lower performance returns. Hypothetical performance has inherent limitations, and investors may experience investment results materially different from those portrayed.

Any forward-looking statements or estimates are based on assumptions, and actual returns are expected to vary from any such statements or estimates. No reliance should be placed on any such statements or forecasts when making any investment decision. The assumptions and projections displayed are estimates, hypothetical in nature, and meant to serve solely as a guideline. The results and analysis are not guarantees of future results because they are derived from mathematical modeling techniques of the economic and financial markets that may or may not reflect actual conditions and events.

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The value added from the tax-advantaged equity strategy reflect Monte Carlo simulations based on the following assumptions:

- Time Horizon -- 10 years
- Individual Stock Volatility -- 41%
- Dividend Yield -- 2.0%
- Bid/Ask Spread (round-trip) -- 0.08%
- Annual Delisting from Index -- 4.0%
- Per-share Commissions -- $0.01
- Expected Market Return -- 7.0%

Due to the complexity of tax law, not every single taxpayer will face the situations described herein exactly as calculated or stated, i.e. the examples and calculations are intended to be representative of some but not all taxpayers. Since each investor’s situation may be different in terms of income tax, estate tax and asset allocation, there may be situations in which the recommendations would not apply. Please discuss any individual situation with tax and investment advisors first before proceeding. Taxpayers paying lower tax rates than those assumed or without taxable income would earn smaller tax benefits from tax-advantaged indexing or even none at all compared to those described.

Index Descriptions

Index Benchmark	Description
DJ Credit Suisse Hedge Fund USD	The Credit Suisse AllHedge Indexes are designed to provide transparent, representative and objective benchmarks of the ten style-based investment strategies of the hedge fund universe. The unique features of the indexes enable them to serve as the underlying for a broad suite of investment products. The composite index of the family is the Credit Suisse AllHedge Index. The sub-indexes covering the individual investment strategies are known collectively as the Credit Suisse AllHedge Strategy Indexes.
HFRI Fund Weighted Composite Index	• Includes over 2200 constituent funds • Includes both domestic and offshore funds • Equal-weighted Index • All funds report assets in USD • No Fund of Funds included in Index • All funds report Net of All Fees returns on a monthly basis • Have at least $50 Million under management or have been actively trading for at least twelve (12) months
MSCI ACWI NR USD	The MSCI ACWI Index is a free float-adjusted market capitalization weighted index that is designed to measure the equity market performance of developed and emerging markets. The MSCI ACWI consists of 44 country indexes comprising 23 developed and 21 emerging market country indexes. The developed market country indexes included are: Australia, Austria, Belgium, Canada, Denmark, Finland, France, Germany, Hong Kong, Ireland, Israel, Italy, Japan, Netherlands, New Zealand, Norway, Portugal, Singapore, Spain, Sweden, Switzerland, the United Kingdom and the United States. The emerging market country indexes included are: Brazil, Chile, China, Colombia, Czech Republic, Egypt, Greece, Hungary, India, Indonesia, Korea, Malaysia, Mexico, Peru, Philippines, Poland, Russia, South Africa, Taiwan, Thailand, and Turkey.
Barclays Aggregate Bond TR	The Barclays US Aggregate Bond Index is a broad-based flagship benchmark that measures the investment grade, US dollar-denominated, fixed-rate taxable bond market. The index includes Treasuries, government-related and corporate securities, MBS (agency fixed-rate and hybrid ARM pass-throughs), ABS and CMBS (agency and non-agency).
Barclays Municipal	The U.S. Municipal Index covers the USD-denominated long-term tax exempt bond market. The index has four main sectors: state and local general obligation bonds, revenue bonds, insured bonds, and prerefunded bonds.
Goldman Sachs Natural Resources	The Goldman Sachs Natural Resources index is a weighted index of 112 stocks from the energy, precious metals and timber sectors.
Russell 2000 TR	The Russell 2000 Index measures the performance of the small-cap segment of the U.S. equity universe. The Russell 2000 Index is a subset of the Russell 3000® Index representing approximately 10% of the total market capitalization of that index. It includes approximately 2000 of the smallest securities based on a combination of their market cap and current index membership. The Russell 2000 is constructed to provide a comprehensive and unbiased small-cap barometer and is completely reconstituted annually to ensure larger stocks do not distort the performance and characteristics of the true small-cap opportunity set.
DJ US Real Estate TR USD	The index's objective is to represent Real Estate Investment Trusts (REIT) and other companies that invest directly or indirectly in real estate through development, management or ownership, including property agencies. The index is a subset of the Dow Jones U.S. Index, which covers 95% of U.S. securities based on float-adjusted market capitalization.
ML US Treasury Bills 0-3 Mon TR USD	The Merrill Lynch 3-Month U.S. Treasury Bill Index is an unweighted index which measures the performance of three-month maturity U.S. Treasury Bills. Each month a one-bill portfolio containing the shortest-term bill having not less than three months to maturity is constructed. To measure holding period returns for the one-bill portfolio, the bill is priced as of the last trading day of the previous month-end and as of the last trading day of the current month.

Term Definitions

Correlation: A statistical measure of how two securities move in relation to each other.

Correlation Coefficient: A measure that determines the degree to which two variable's movements are associated.

Monte Carlo Simulation: A problem-solving technique used to approximate the probability of certain outcomes by running multiple trial runs, called simulations, using random variables.